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Mass transfer in cross flow of non-Newtonian fluid around a circular cylinder

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INTRODUCTION

NUMEROUS situations of practical interest require a knowledge of heat and mass transfer from cylinders subjected to external flow of non-Newtonian fluids. Although some work has been reported on the flow of viscoelastic fluids around circular cylinders, experimental work on heat and mass transfer is very meagre. It is limited only to experimental measurements of heat transfer by Shah *et al.* [1], James and Acosta [2], Mizushina *et al.* [3, 4] and Takahashi *et al.* [5], and of mass transfer by Luikov *et al.* [6] and Mizushina *et al.* [3]. Most of the results presented so far are confined to the high Reynolds number region. This investigation extends the experimental data to the low Reynolds number region and compares the effectiveness of power-law and Prandtl–Eyring models in predicting heat and mass transfer data.

THEORETICAL BACKGROUND AND LITERATURE REVIEW

Forced convection

Acrivos and coworkers [1, 7, 8] presented an approximate solution for flow of a pseudoplastic fluid of infinitely large Prandtl number. Bizzel and Slattery [9] extended the Karman–Pohlhausen solution to power-law fluid. Wolf and Szewczyk [10] obtained a Blasius series solution for pseudoplastic fluid. Mizushina and Usui [11] extended Karman–Pohlhausen–Dienemann [12] approximate solutions to non-Newtonian fluids using both power-law and Prandtl–Eyring models. Their results revealed that the power-law model suffers from 'zero defect'.

For power-law fluid obeying

$$\tau_w = K\left(\frac{du}{dy}\right)^n \tag{1}$$

Acrivos and coworkers [1, 7, 8], for laminar forced convection, obtained

$$Nu = f(n)Re_p^{1/(n+1)}Pr_A^{1/3}. \tag{2}$$

Using their own data, Mizushina and coworkers [3, 11] showed that for power-law fluids equation (2) can be expressed as

$$Nu = 0.72n^{-0.4}Re_p^{1/(n+1)}Pr_A^{1/3}. \tag{3}$$

For a pseudoplastic fluid obeying the Prandtl–Eyring model

$$\tau_w = A \operatorname{arcsinh}\left(\frac{1}{B}\frac{du}{dy}\right) \tag{4}$$

they showed that

$$Nu = 0.87Re_E^{0.48}Pr_E^{1/3}. \tag{5}$$

James and coworkers [2, 13] reported that at low velocities, the drag coefficient and heat transfer coefficient for dilute polymeric solutions flowing laminarly past cylinders are identical to those for Newtonian fluids but have asymptotic values for sufficiently high velocities. James and Acosta [2] attributed it to the viscoelastic behaviour. Metzner and Astarita [14] assumed solid-like behaviour in high Deborah number flows and explained it using the concept of boundary-layer thickening. Mashelkar and Marrucci [15] proposed the concept of an elastic boundary layer for explaining such anomalous transport phenomena. Ruckenstein and Ramgopal [16] used a boundary-layer approach to explain this anomalous behaviour. They obtained a general equaion of the form

$$NuPr^{-1/3}[(De'+0.25)/Re]^{1/2}$$
$$= 0.0897(De'+0.25)\{[4(9/(De'+0.25)+1)^{3/4}-1]^{2/3}-3^{2/3}\} \tag{6}$$

for the higher Reynolds number region. They arbitrarily modified the coefficients and exponents to get a relation

$$NuPr^{-1/3}[(De'+0.25)/Re]^{0.28}$$
$$= 0.1489(De'+0.25)\{[36(1/(De'+0.25)+1)^{3/4}-33]^{2/3}-3^{2/3}\} \tag{7}$$

for the lower Reynolds number region and in better agreement with the results of James and Acosta [2]. For fluids with negligible elastic effects, i.e. $De' = 0$, equation (7) reduces to

$$NuPr^{-1/3} = 0.966Re^{0.28} \tag{8}$$

which is a Newtonian relation.

Kumar *et al.* [17] simply used the Newtonian and non-Newtonian Reynolds numbers and defined an effective viscosity as

$$\mu_e = K(D/U_\infty)^{(1-n)}. \tag{9}$$

By substituting it in place of μ in various dimensionless groups for Newtonian fluid they obtained the corresponding groups for power-law fluid.

Luikov *et al.* [6, 18] correlated their mass transfer results by

$$Sh = C\,Re_p^m Sc_A^{1/3} \tag{10}$$

where $C = 0.31, m = 0.52$ for $n = 1$ and $C = 0.5, m = 0.39$ for $n = 0.88$. Takahashi *et al.* [5] used Acrivos' approach and obtained a single-valued correlation

$$Nu = 1.3Re_p^{0.35}Pr_A^{1/3} \tag{11}$$

for $0.78 < n < 1.0$.

Natural convection

The situation of laminar natural convection heat transfer from a single cylinder, sphere, etc. to power-law fluids was analysed by Acrivos [19]. Mass transfer equivalents of his asymptotic solution of the boundary-layer equation may be written as

$$Sh_0 = A'Gr_{Am}'^{1/[2(n+1)]}Sc_A'^{n/(3n+1)} \tag{12}$$

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NOMENCLATURE

- A rheological parameter of Prandtl–Eyring model [$kg\ m^{-1}\ s^{-2}$]
- A' constant
- B rheological parameter of Prandtl–Eyring model [s^{-1}]
- B' constant
- C constant
- C_p heat capacity of fluid [$J\ kg^{-1}\ K^{-1}$]
- D cylinder diameter [m]
- De Deborah number, $B'\bar{\theta}U_\infty/D$
- De' modified Deborah number, $2000U_\infty\bar{\theta}/D$
- D_m diffusivity [$m^2\ s^{-1}$]
- Gr'_{Ah} Grashof number defined by Acrivos for power-law fluids for heat transfer, $\rho_f^2 D^{n+2}(\beta g\Delta\theta)^{2-n}/K^2$
- Gr'_{Am} Grashof number defined by Acrivos for power-law fluids for mass transfer, equation (13)
- g acceleration due to gravity [$m\ s^{-2}$]
- K consistency index of power-law model [$kg\ s^{n-2}\ m^{-1}$]
- K_b consistency index at bulk temperature [$kg\ s^{n-2}\ m^{-1}$]
- K_c mass transfer coefficient [$m\ s^{-1}$]
- K_s consistency index at surface temperature [$kg\ s^{n-2}\ m^{-1}$]
- L length [m]
- m constant
- n flow behaviour index of power-law model
- Nu Nusselt number, hD/λ
- Pe Péclet number, $DU_\infty\rho C_p/\lambda$
- Pr Newtonian Prandtl number, $C_p\mu/\lambda$
- Pr_A generalized Prandtl number, $C_pDU_\infty Re_p^{-2/(n+1)}/\lambda$ or $U_\infty D\ Re_p^{-2/(n+1)}/D_m$
- Pr'_A Prandtl number defined by Acrivos for natural convection, $(\rho_f C_p/\lambda)(K/\rho_f)^{2/(1+n)}D^{(1-n)/(1+n)} \times (D\beta g\Delta\theta)^{[3(n-1)]/[2(n+1)]}$
- Pr_E Prandtl number for Prandtl–Eyring model, $C_p(A/B)/\lambda$ or $(A/B)/\rho D_m$
- Re Newtonian Reynolds number, $DU_\infty\rho/\mu$
- Re_E Reynolds number for Prandtl–Eyring model, $DU_\infty\rho/(A/B)$
- Re_P Reynolds number for power-law model, $D^n U_\infty^{2-n}\rho/K$
- Sc Schmidt number, $\mu_e/\rho D_m$
- Sc_A Schmidt number for power-law model based on Acrivos approach $U_\infty D\ Re_p^{-2/(n+1)}/D_m$
- Sc'_A Schmidt number defined by Acrivos for natural convection, equation (14)
- Sc_E Schmidt number for Prandtl–Eyring model, $(A/B)/\rho D_m$
- Sh Sherwood number, K_cD/D_m
- Sh_0 Sherwood number for natural convection, equation (12)
- U_∞ free-stream velocity [$m\ s^{-1}$].

Greek symbols

- β coefficient of volumetric expansion [K^{-1}]
- β' parameter defined by equation (22)
- λ thermal conductivity of solution [$W\ m^{-1}\ K^{-1}$]
- μ Newtonian viscosity [$N\ s\ m^{-2}$]
- μ_e effective viscosity [$N\ s\ m^{-2}$]
- ρ density [$kg\ m^{-3}$]
- ρ_f density of fluid [$kg\ m^{-3}$]
- $\Delta\rho$ density difference [$kg\ m^{-3}$]
- σ parameter defined by equation (16)
- $\Delta\theta$ temperature difference [K]
- $\bar{\theta}$ relaxation time (s)
- τ_w shear stress [$N\ m^{-2}$].

for

$$Gr'^{1/(n+1)}_{Am}Sc'^{1/(3n+1)}_A > 36, \quad Sc'_A > 10$$

where

$$Gr'_{Am} = \frac{\rho_f^2 D^{n+2}(g\Delta\rho)^{2-n}}{K^2\rho_f^{2-n}} \tag{13}$$

and

$$Sc'_A = \left(\frac{K}{\rho_f}\right)^{2/(1+n)} D^{(1-n)/(1+n)} \times \left(Dg\frac{\Delta\rho}{\rho_f}\right)^{[3(n-1)]/[2(1+m)]} \frac{1}{D_m}. \tag{14}$$

The constant A' is approximately equal to 0.55 and is a function of both n and characteristic dimension D. For cylinders, where the characteristic dimension used by Acrivos is cylinder radius, A' varies from 0.36 to 0.45 for $0.1 < n < 1.5$ [20]. The only reported experimental work on natural convective contribution during external flow of non-Newtonian fluids is that of Yamanaka and Mitsuishi [21] who used Acrivos' approach in treating their results for heat transfer from spheres to 2.61% aqueous methyl cellulose, 5.5% aqueous CMC, 0.74% aqueous sodium polyacrylate and 1.48% polythylene oxide. Their asymptotic equation

$$Nu-2 = [(0.866\sigma^{2/3}Pe^{1/3}-0.553\sigma-0.341)^{3/2} + (0.44Gr'^{1/[2(n+1)]}_{Ah}Pr'^{n/(3n+1)}_A)^{3/2}]^{2/3}\left(\frac{K_b}{K_s}\right)^{1/(3n+1)} \tag{15}$$

where

$$\sigma = -2.475n^3+6.738n^2-7.868n+4.740 \tag{16}$$

correlated the data within an average deviation of $\pm 29.3\%$. This correlation was recommended to be used within the range of

$$1.0 < Pe < 10^3, \quad 1.6\times10^{-6} < Gr'_{Ah} < 0.44, \quad 2.6\times10^4 < Pr'_A < 6.4\times10^5$$

and

$$0.24 < Gr'^{1/(n+1)}_{Ah}Pr'^{2n/(3n+1)}_A < 74.$$

EXPERIMENTAL

The experimental set-up and procedure employed was quite similar to that used earlier [22, 23]. A test cylinder weighed to the nearest 0.05 mg was mounted in a 10.0-cm-diam. Pyrex glass test column and was allowed to come in contact with the test fluid flowing at a known flow rate for a known interval of time. The resultant weight loss of the test specimen was used to calculate mass transfer rate.

Test cylinders were prepared by compressing chemically pure benzoic acid (Sarabhai Merck, Baroda, India) in cylindrical moulds. The two ends of all cylinders used were masked with wax. Test fluids used were demineralised water and 0.5, 0.75, 1.0 and 1.5% aqueous solutions of H-V grade CMC powder (Robert Johnson, Bombay). The flow curves and rheological parameters of these solutions were determined either by a capillary tube viscometer or Synchro-Lectric viscometer. Table 1 gives the rheological parameters, density and molecular diffusivity for all the test fluids used in this work.

The diffusivity of benzoic acid in aqueous CMC solution

Table 1. Physical properties of the test fluids

Fluid	Temperature (°C)	n	K	ρ	A/B	$D_m \times 10^6$
Water	30.0	1	0.00801	0.9956	—	9.476
	35.0	1	0.00724	0.9943	—	11.36
0.5% CMC	32.5	0.97	0.0267	1.002	0.027	9.144
0.75% CMC	32.5	0.92	0.302	1.003	0.31	8.808
1.0% CMC	30.0	0.94	2.20	1.002	3.20	8.881
1.5% CMC	30.0	0.89	7.10	1.003	8.90	8.399

was determined by the rotating disk technique; its solubility by the 'equilibrated solution' method. The details of these techniques are reported elsewhere [24, 25]. Physical properties for demineralised water were taken to be those of pure water [22].

RESULTS AND DISCUSSION

The mass transfer coefficient k_c exhibits the usual dependence on cylinder diameter, i.e. it decreases with increasing cylinder diameter. The L/D ratio, however, does not have any significant influence on k_c. For further analysis, heat and mass transfer data of Mizushina *et al.* [3] have also been included. The Reynolds number range (Re_p = 30–8000) covered by Mizushina *et al.* for non-Newtonian fluids is in the higher laminar region and that of present work (Re_p = 0.0018–513) is in the lower (creeping flow) region.

Mizushina *et al.* [3] showed that $Nu(\text{or } Sh) \propto n^{-0.4}$ and included it in their correlation, i.e. equation (3). This equation correlates their data with an average deviation of ±7.8% (r.m.s. deviation = 9.4). Their results, however, do not show any appreciable scatter by neglecting the term $n^{-0.4}$ and can be correlated by

$$Nu(\text{or } Sh) = 0.759 Re_p^{1/2} (Pr_A \text{ or } Sc_A)^{1/3}. \quad (17)$$

In Fig. 1 their results are shown as $Nu(\text{or } Sh)(Pr \text{ or } Sc)^{-1/3}$ vs Re_p plot, where Pr(or Sc) is based on the effective viscosity defined by equation (9). The regression analysis shows that

$$Nu(\text{or } Sh) = 0.7851 Re_p^{0.5} (Pr \text{ or } Sc)^{1/3} \quad (18)$$

correlates the results with an average deviation of ±7.5% (r.m.s. deviation = 8.5). The Prandtl–Eyring model-based equation (5) correlates these results with a deviation of ±5.8% (r.m.s. deviation = 6.7).

A comparison of the average and r.m.s. deviations indicates that all the three approaches discussed above are more or less equally successful in correlating the higher laminar regime (Reynolds number ⩾ 10) data. The least deviation, however, is obtained with the Prandtl–Eyring approach which indicates that for moderate non-Newtonian behaviour this model appears to be the most effective and simple approach of correlating the higher laminar regime heat and mass transfer data.

Most of the data obtained in the present work are in the creeping flow regime. In this regime the Stokes or Oseen analysis has been applied to Newtonian heat transfer by several workers [26, 27]. Hilpert [26] suggested a relation for air as

$$Nu = 0.89 Re^{1/3}, \quad 0.1 < Re < 100 \quad (19)$$

which for fluids of arbitrary Prandtl numbers could be written as

$$Nu = 1.002 Re^{1/3} Pr^{1/3}. \quad (20)$$

Oseen's analysis of heat transfer in creeping flow as presented by Tomotika and Yoshida [28] resulted in

$$Nu = \beta' - \frac{Pr^2 Re^2}{12}(1+\beta'^2) \quad (21)$$

where

$$\beta' = \frac{2}{\ln(8/Pr \cdot Re) - 0.577}. \quad (22)$$

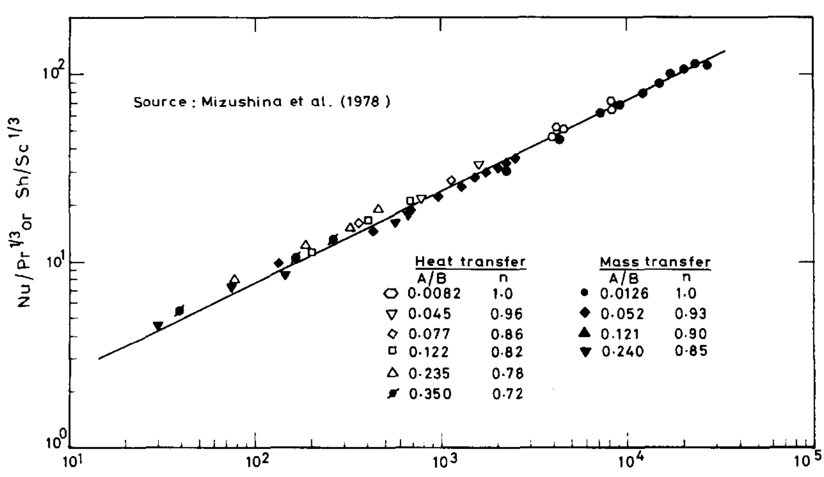

FIG. 1. Power-law model (effective viscosity approach) based correlation of heat and mass transfer data of Mizushina *et al.* [3].

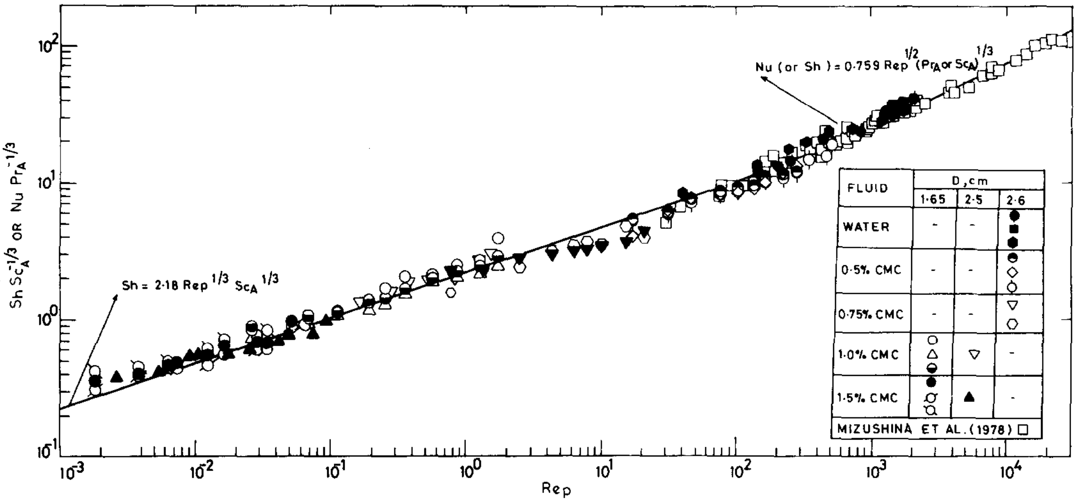

FIG. 2. Power-law model based correlation of heat and mass transfer data.

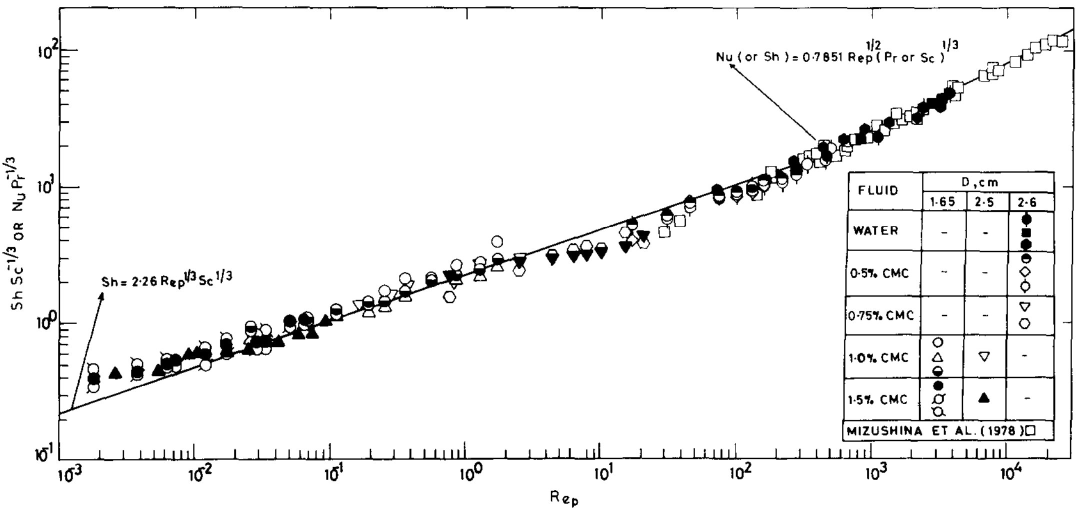

FIG. 3. Power-law model (effective viscosity approach) based correlation of heat and mass transfer data.

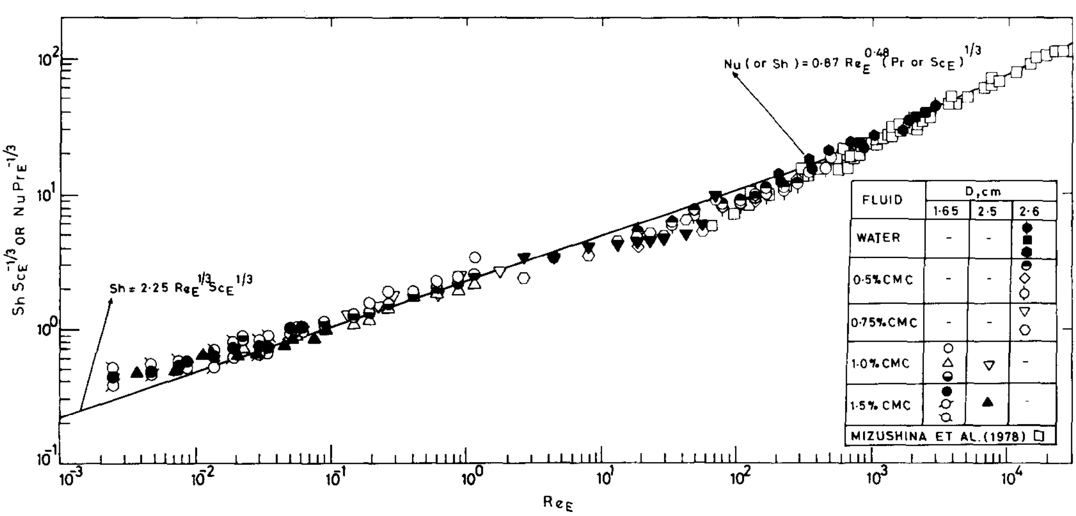

FIG. 4. Prandtl–Eyring model based correlation of heat and mass transfer data.

Table 2. Natural convective contribution (cylinder diam. = 1.65 cm)

Model	Equation	1.0% Aq. CMC			1.5% Aq. CMC		
		Gr	Sc	Sh	Gr	Sc	Sh
Acrivos	$Sh = 0.8Gr'^{1/[2(n+1)]}_{Am}Sc'^{n/(3n+1)}_{A}$	0.67	2.5×10^5	11.7	0.014	9.3×10^5	8.5
Effective viscosity	$Sh = 0.53(Gr\,Sc)^{1/4}$	0.132	2.4×10^5–3.1×10^5	10.2	0.04	1.0×10^6–1.4×10^6	7.3
Prandtl–Eyring	$Sh_0 = 0.53(Gr_E Sc_E)^{1/4}$	0.490	3.6×10^5	10.9	0.069	1.05×10^6	8.7

For low Prandtl numbers ($Pr = 0.7$) equation (21) breaks down at $Re \simeq 6$ whereas for high Prandtl numbers ($Pr > 500$) it does so at Reynolds numbers as low as 0.001. Thus it cannot be applied to viscous non-Newtonian fluids where Prandtl of Schmidt numbers are very large.

Present results along with the data of Mizushina *et al.* [3] are shown in Figs. 2–4 as (Sherwood number)(Schmidt number)$^{-1/3}$ vs Reynolds number plots with dimensionless groups defined using effective viscosity, Acrivos and Prandtl–Eyring approaches, respectively. From these figures it can be concluded that for $Re_p < 10$ (or $Re_E < 10$)

$$Sh \propto (Re_p \text{ or } Re_E)^{1/3}(Sc \text{ or } Sc_A \text{ or } Sc_E)^{1/3} \quad (23)$$

irrespective of the approach used in defining these dimensionless numbers. The data in each case show gradual transition from the creeping to the higher laminar flow regime as observed for Newtonian fluids [29]. Further, the similarity in the extent of scatter of the data points indicates that, for mild non-Newtonian behaviour, in this regime all the three rheological approaches are more or less equally successful in correlating the experimental data.

The asymptotic behaviour of heat transfer coefficient as reported by James and Acosta [2] is not observed in the present work. This is largely due to the use of highly concentrated polymer solutions (~ 7500–30,000 p.p.m.), larger diameter cylinders and low flow velocities. Under these conditions elastic forces are negligible. From Figs. 2–4 it is seen that the data for 0.75% CMC solution do show an asymptotic trend, however, these are too limited to draw any definite conclusion.

The regression analysis indicated that creeping flow regime (Re_p or $Re_E \leqslant 10$) data could be represented by

$$Sh = A(Re_p \text{ or } Re_E)^{1/3}(Sc_A \text{ or } Sc \text{ or } Sc_E)^{1/3} \quad (24)$$

where A is 2.18 for the Acrivos approach, 2.26 for the effective viscosity approach and 2.25 for the Prandtl–Eyring model. The corresponding higher Reynolds number regime data can be correlated by equations (17), (18), and (5), respectively.

Figures 2–4 all show that natural convective contribution is significant for Re_p(or Re_E) < 0.01. The magnitude of these contribution as calculated from the various equations for 1.0 and 1.5% CMC solutions (Table 2) are of comparable magnitude and are less than 10% of the total values of Sherwood numbers. Due to this no attempt has been made to account for natural convective contribution in the mass transfer correlations.

It would also be appropriate to point out that Soehngen [30] observed that—at very low Reynolds number—viscosity plays a dominant role and boundary layers become comparable to body dimensions. Under such a situation boundary-layer theory does not hold good and the use of classical dimensionless parameters for representing data should be made with caution. These conclusions, however, are based on data obtained with thin wires ($D \leqslant 0.5$ mm) and may not be true for large diameter cylinders as used in the present work.

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